(1.) $2x - y - 4z = 3$

$-x + 3y + z = -10$

$3x + 2y - 2z = -2$ here is the problem

$-3x + 9y + 3z = -30$ multiply eq 2 thru by 3

$11y + z = -32$ add equations

$-2x + 6x + 2z = -20$ multiply eq 2 thru by 2

$2x - y - 4z = 3$ put this here

$5y - 2z = -17$ add equations

$22y + 2z = -64$ multiply 11y+z=-32 thru by 2

$27y = -81$ add equations

$\frac{27y}{27} = \frac{-81}{27}$ divide each side by 27

$y = -3$ divide and cancel

$11(-3) + z = -32$ replace y with -3

$-33 + z = -32$ multiply

$+33 \quad +33$ add 33 to each side

$z = 1$ add

$-x + 3y + z = -10$ use this equation to find x

$-x + 3(-3) + 1 = -10$ replace y and z with -3 & 1

$-x - 9 + 1 = -10$ multiply

$-x - 8 = -10$ combine like terms

$x + 8 = 10$ multiply thru by -1

$-8 \quad -8$ subtract 8 from each side

x = 2 subtract

results: x = 2 ; y = -3 ; z = 1

(2.) 2x - y + 4z = 1

x - y + z = 0

x + y + z = 1 here is the problem

2x + 2z = 1 add equations

2x - y + 4z = 1 put this here

x + y +z = 1 put this here

3x + 5z = 2 add equations

6x + 6z = 3 multiply 2x + 2z = 1 thru by 3

-6x - 10z = -4 multiply 3x + 5z = 2 thru by -2

-4z = -1 add equations

$\frac{-4z}{-4} = \frac{-1}{-4}$ divide each side by -4

z = 1/4 cancel

2x + 2(1/4) = 1 replace z with 1/4

2x + 0.5 = 1 multiply

- 0.5 -0.5 subtract 0.5 from each side

2x = 0.5 subtract

x = 1/4 multiply each side by 1/2, cancel

x + y + z = 1 use this equation to find y

(1/4) + y + (1/4) = 1 replace x and z with 1/4 and 1/4

y + (1/2) = 1 combine like terms

-1/2 -1/2   subtract 1/2 from each side

y = 1/2 subtract

results: x = 1/4 ; y = 1/2 ; z = 1/4

(3.) 2x + 3y - 4z = -8

x + y - 2z = -5

7x - 2y + 5z = 4 here is the problem

2x + 2y - 4z = -10 multiply eq 2 thru by2

7x - 2y + 5z = 4 put this here

9x + z = -6 add equations

2x + 3y - 4z = -8 put this here

-3x - 3y + 6z = 15 multiply eq 2 thru by -3

-x + 2z = 7 add equations

-9x + 18z = 63 multiply that equation thru by 9

9z + z = -6 put this here

19z = 57 add equations

19 19 divide each side by 19

z = 3 divide and cancel

9x + 3 = -6 replace z with 3

- 3 -3 subtract 3 from each side

9x = -9 subtract

9 9 divide each side by 9

x = -1 divide and cancel

x + y - 2z = -5 use this equation to find y

-1 + y - 2(3) = -5 replace x and z with -1 and 3

-1 + y - 6 = -5 multiply

y - 7 = -5 combine like terms

\+ 7 +7 add 7 to each side

_____

y = 2 add

results: x = -1 ; y = 2 ; z = 3

(4.) 3x + 4y - z = -2

2x - 3y + z = 4

x - 6y + 2z = 5 here is the problem

-x + 6y - 2z = -5 multiply eq 3 thru by -1

4x - 6y + 2z = 8 multiply eq 2 thru by 2

_____

3x = 3 add equations

___ ___

3 3 divide each side by 3

x = 1 divide and cancel

3x + 4y - z = -2

2x - 3y + z = 4 put these two equations here

_____

5x + y = 2 add equations

5(1) + y = 2 replace x with 1

5 + y = 2 multiply

-5 -5 subtract 5 from each side

_____

y = -3 subtracte

2x - 3y + z = 4 put this equation here

2(1) - 3(-3) + z = 4 replace x & y with 1 & -3

2 + 9 + z = 4 multiply

11 + z = 4 combine like terms

-11 -11 subtract 11 from each side

z = -7 subtract

results: x = 1 ; y = -3; z = -7

(5.) 5x + y - z = 9

3x + y + 2z = 17

x + 2y + 3z = 20 here is the problem

5x + y - z = 9

3x + y + 2z = 17 put these two equations here

2x - 3z = -8 subtract equations

6x + 2y + 4z = 34 multiply eq 2 thru by 2

-x - 2y - 3z = -20 multiply eq 3 thru by -1

5x + z = 14 subtract equations

15x + 3z = 42 multiply that equation thru by 3

2x - 3z = -8 put this equation here

17x = 34 add equations

$\frac{17x}{17} = \frac{34}{17}$ divide each side by 17

x = 2 divide and cancel

5(2) + z = 14 replace x with 2

10 + z = 14 multiply

-10 -10 subtract 10 from each side

z = 4 subtract

3x + y + 2z = 17 put this equation here

3(2) + y + 2(4) = 17 replace x & z with 2 & 4

6 + y + 8 = 17 multiply

y + 14 = 17 combine like terms

-14 -14 subtract 14 from each side

y = 3 subtract

results: x = 2; y = 3; z = 4

(6.) 2a - 3b + c = 2

3a + 2b - c = 4

2a - 3b + c = 5 here is the problem

5a - b = 9 add equations

2a - 3b + c = 2

3a + 2b - c = 4 put these two equations here

5a - b = 6 add equations

-5a + b = -6 multiply that equation thru by -1

5a - b = 9 put this equation here

0 = 3 add equations

result: no solution

(7.) 2x + 3y + 4z = 8

4x + 9y + 8z = 17

6x + 12y + 16z = 31 here is the problem

4x + 9y + 8z = 17 put this here

-4x - 6y - 8z = -16 multiply eq 1 thru by -2

3y = 1 add equations

$\frac{3y}{3} = \frac{1}{3}$ divide each side by 3

y = 1/3 cancel

8x + 12y + 16z = 32 multiply eq 1 thru by 4

6x + 12y + 16z = 31 put this equation here

2x = 1 subtract equations

$\frac{2x}{2} = \frac{1}{2}$ divide each side by 2

x = 1/2 cancel

4(1/2) + 9(1/3) + 8z = 17 replace x & y with 1/2 & 1/3

2 + 3 + 8z = 17 multiply

8z + 5 = 17 combine like terms

-5 -5 subtract 5 from each side

8z = 12 subtract

$\frac{8z}{8} = \frac{12}{8}$ divide each side by 8

z = 1.5 divide and cancel

results: x = 1/2 ; y = 1/3; z = 1.5

(8.) 2x + 3y = -2

4y + 2z = -10

3x + 5z = 1 here is the problem

2x + 3y = -2 put this here

$\frac{2}{2} \quad \frac{2}{2} \quad \frac{-2}{2}$ divide thru by 2

x + 1.5y = -1 divide and cancel

-1.5y - 1.5y subtract 1.5y from each side

x = -1 - 1.5y subtract

3(-1 - 1.5y) + 5z = 1 replace x with -1 - 1.5y

-3 - 4.5y + 5z = 1 multiply thru parentheses

+ 3 + 3 add 3 to each side

-4.5y + 5z = 4 add

-9y + 10z = 8 multiply that eq thru by 2

20y + 10z = -50 multiply eq 2 thru by 5

-29y = 58 subtract equations

$\frac{-29y}{-29} = \frac{58}{-29}$ divide each side by -29

y = -2 divide and cancel

x = -1 - 1.5(-2) replace y with -2

x = -1 + 3 multiply

x = 2 add

$3x + 5z = 1$

$3(2) + 5z = 1$ replace x with 2

$6 + 5z = 1$ multiply

$-6 \quad -6$ subtract 6 from each side

$5z = -5$ subtract

$\frac{5z}{5} = \frac{-5}{5}$ divide each side by 5

$z = -1$

results: $x = 2$; $y = -2$; $z = -1$

(9.) $2x - y + 3z = -9$

$x + 3y - z = 10$

$3x + y - z = 8$ here is the problem

$-x - 3y + z = -10$ multiply eq 2 thru by -1

$2x - 2y = -2$ add equations

$3x + 9y - 3z = 30$ multiply eq 2 thru by 3

$2x - y + 3z = -9$ put this here

$5x + 8y = 21$ add equations

$8x - 8y = -8$ multiply $2x - 2y = -6$ thru by 4

$13x = 13$ add equations

$\frac{13x}{13} = \frac{13}{13}$ divide each side by 13

$x = 1$ divide and cancel

$2(1) - 2y = -2$ replace x with 1

$2 - 2y = -2$ multiply

$-2 + 2y = 2$ multiply thru by -1

$+2 \quad + \quad 2$ add 2 to each side

$2y = 4$ add

$\frac{2y}{2} = \frac{4}{2}$ divide each side by 2

$y = 2$ divide and cancel

$x + 3y - z = 10$

$1 + 3(2) - z = 10$ replace x & y with 1 & 2

$1 + 6 - z = 10$ multiply

$7 - z = 10$ combine like terms

$-7 + z = -10$ multiply thru by -1

$+7 \quad + \quad 7$ add 7 to each side

$z = -3$ add

results: $x = 1$; $y = 2$; $z = -3$

(10.) $3x + 4z = 22$

$2y - z = 2$

$5x + 3y = 19$ here is the problem

$-2y + z = -2$ multiply eq 2 thru by -1

$+ \quad 2y \quad + 2y$ add 2y to each side

$z = 2y - 2$ add

$3x + 4(2y - 2) = 22$ replace z with 2y - 2

$3x + 8y - 8 = 22$ multiply thru

$$
\begin{array}{rl}
+\,8 \quad +\,8 & \text{add 8 to each side} \\
\hline
3x + 8y = 30 & \text{add} \\
5x + 3y = 19 & \text{put this here} \\
15x + 40y = 150 & \text{multiply } 3x+8y=30 \text{ thru by 5} \\
15x + 9y = 57 & \text{multiply } 5x + 3y = 19 \text{ thru by 3} \\
\hline
31y = 93 & \text{subtract equations} \\
\frac{31}{31} \quad \frac{93}{31} & \text{divide each side by 31} \\
y = 3 & \text{divide and cancel}
\end{array}
$$

$$
\begin{array}{rl}
3x + 8(3) = 30 & \text{replace y with 3} \\
3x + 24 = 30 & \text{multiply} \\
-24 \quad -24 & \text{subtract 24 from each side} \\
\hline
3x = 6 & \text{subtract} \\
\frac{3x}{3} = \frac{6}{3} & \text{divide each side by 3} \\
x = 2 & \text{divide and cancel}
\end{array}
$$

$$
\begin{array}{rl}
2(3) - z = 2 & \text{replace y with 3} \\
6 - z = 2 & \text{multiply} \\
-6 + z = -2 & \text{multiply thru by -1} \\
+\,6 \quad +\,6 & \text{add 6 to each side} \\
\hline
z = 4 & \text{add}
\end{array}
$$

results: $x = 2$; $y = 3$; $z = 4$