Simultaneous Quadratics

1. Two cubical coal bins together hold 280 cubic feet of coal, and the sum of their lengths is 10 feet. Find the length of each bin.

Let x = the length of the 1st bin

Let y = the length of the 2nd bin

$x + y = 10$

$x^3 + y^3 = 280$ here is the problem

$y = 10 - x$ subtract x from each side of $x + y = 10$

$x^3 + (10 - x)^3 = 280$ replace y with 10 - x

$x^3 + 10^3 - 300x + 30x^2 - x^3 = 280$ use the binomial theorem

$30x^2 - 300x + 1000 = 280$ combine like terms

$\frac{30x^2}{10} - \frac{300x}{10} + \frac{1000}{10} = \frac{280}{10}$ divide thru by 10

$3x^2 - 30x + 100 = 28$ divide

$-28 \quad -28$ subtract 28 from each side

$3x^2 - 30x + 72 = 0$ subtract

$\frac{3x^2}{3} - \frac{30x}{3} + \frac{72}{3} = \frac{0}{3}$ divide thru by 3

$x^2 - 10x + 24 = 0$ divide and cancel

$(x - 6)(x - 4) = 0$ factor

$x - 4 = 0$ set this factor equal to 0

$+ 4 \quad +4$ add 4 to each side

$x = 4$ add

$y = 10 - x$ use this equation to find y

$y = 10 - 4$ replace x with 4

$y = 6$ subtract

results: 4 feet and 6 feet

2. The sum of the radii of two circles is 25 inches, and the difference of their areas is 125pi square inches. Find the radii.

$x + y = 25$

$(pi)x^2 - (pi)y^2 = 125pi$ here is the problem

$x^2 - y^2 = 125$ divide thru by pi, cancel

$y = 25 - x$ subtract x from each side of $x + y = 25$

$x^2 - (25 - x)^2 = 125$ replace y with $25 - x$

$x^2 - 625 + 50x - x^2 = 125$ square the binomial

$50x - 625 = 125$ combine like terms

$+ 625 \quad +625$ add 625 to each side

$50x = 750$ add

$\frac{50x}{50} = \frac{750}{50}$ divide each side by 50

$x = 15$ divide and cancel

$y = 10$

3. The area of a right triangle is 150 square feet, and its hypotenuse is 25 feet. Find the arms of the triangle.

$(1/2)bh = 150$

$b^2 + h^2 = 25^2$ here is the problem

$bh = 300$ multiply each side of $(1/2)bh = 140$ thru by 2

$h = 300/b$ divide each side by b

$b^2 + (300/b)^2 = 625$ make substitution and square 25

$b^4 + 90,000 = 625b^2$ multiply thru by b^2 and cancel

$-625b^2 \quad - 625b^2$ subtract this from each side

$b^4 - 625b^2 + 90,000 = 0$ subtract

$(b^2 - 225)(b^2 - 400) = 0$ factor

$(b - 15)(b + 15)(b - 20)(b + 20) = 0$ factor

$b - 15 = 0$ set this factor equal to 0

$+ 15 \quad +15$ add 15 to each side

$b = 15$ add

$h = 300/15$ replace b with 15

$h = 20$ divide

results: b = 15 and h = 20

4. The combined capacity of two cubical tanks is 637 cubic feet, and the sum of an edge of one and an edge of the other is 13 feet.

(a) Find the length of a diagonal of any face of each cube.

$x + y = 13$

$x^3 + y^3 = 637$ here is the problem

$y = 13 - x$ subtract x from each side

$x^3 + (13 - x)^3 = 637$ replace y with 13 - x

$x^3 + (13)^3 - 3(13)^2(x) + 3(13)(x)^2 - x^3 = 637$

[use the biomial theorem]

$2197 - 507x + 39x^2 = 637$ multiply combine like terms

$39x^2 - 507x + 2197 = 637$ rearrange terms

$- 637 \quad -637$ subtract 637 fr ea side

$39x^2 - 507x + 1560 = 0$ subtract

$\frac{39x^2}{39} - \frac{507x}{39} + \frac{1560}{39} = \frac{0}{39}$ divide thru by 39

$x^2 - 13x + 40 = 0$ divide thru by 39, cancel

$(x - 8)(x - 5) = 0$ factor

$x - 8 = 0 \quad x - 5 = 0$ set each factor equal to 0

$+8 \quad +8 \quad +5 \quad +5$ add this to each side

$x = 8$; $x = 5$ add

results: The diagonals are $8\sqrt{2}$ and $5\sqrt{2}$.

(b) Find the distance from upper left-hand corner to lower right-hand corner in either cube.

$z^2 = a^2 + b^2 + c^2$ use this formula

$z^2 = 8^2 + 8^2 + 8^2$ make substitutions

$z^2 = 192$ square and add

$z^2 = (64)(3)$ factor

$z = 8\sqrt{3}$ take square roots

6. After street improvement it is found that a certain corner rectangular lot

has lost (1/10) of its length and (1/15) of its width. Its

perimeter has been decreased

by 28 feet, and the new area is 3024 square feet.

Find the reduced dimensions of the lot.

Let L = the old Length of the lot.

Let w = the old width of the lot.

$2(9/10)L + 2(14/15)w = 2L + 2w - 28$ here is the perimeter eq

$(9/10)L * (14/15)w = 3024$ here is the area equation

$(18/10)L + (28/15)w = 2L + 2w - 28$ multiply

$(9/5)L + (28/15)w = 2L + 2w - 28$ reduce the fraction

$27L + 28w = 30L + 30w - 420$ multiply thru by 15, cancel

$30L + 30w - 420 = 27L + 28w$ rearrange like this

$+ 420 \quad + 420$ add 420 to each side

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$30L + 30w = 27L + 28w + 420$ add

$- 28w \quad - 28w$ subtract 28w from each side

$$30L + 2w = 27L + 420$$ subtract

$$-27L \quad - 27L$$ subtract 27L from each side

$$3L + 2w = 420$$ subtract

$$-3L \quad - 3L$$ subtract 3L from each side

$$\frac{2w}{2} = \frac{420}{2} - \frac{3L}{2}$$ subtract / divide thru by 2

$$w = 210 - 1.5L$$ divide and cancel

$$(9/10)L * (14/15)w = 3024$$ here is the area equation

$$(3/5)(7/5)Lw = 3024$$ reduce the fractions

$$(21/25)Lw = 3024$$ multiply fractions

$$21Lw = 75,600$$ multiply each side by 25 and cancel

$$\frac{21Lw}{21} = \frac{75,600}{21}$$ divide each side by 21

$$Lw = 3600$$ divide and cancel

$$(L)(210 - 1.5L) = 3600$$ replace w with 210 - 1.5L

$$210L - 1.5L^2 = 3600$$ multiply thru parentheses

$$-210L + 1.5L^2 = -3600$$ multiply thru by -1

$$1.5L^2 - 210L = -3600$$ rearrange terms

$$+ \quad 3600 \quad +3600$$ add 3600 to each side

$$1.5L^2 - 210L + 3600 = 0$$ add

$$\frac{1.5L^2}{1.5} - \frac{210L}{1.5} + \frac{3600}{1.5} = \frac{0}{1.5}$$ divide thru by 1.5

$$L^2 - 140L + 2400 = 0$$ divide and cancel

$$(L - 120)(L - 20) = 0$$ factor

$L - 120 = 0$ set this factor equal to 0

$+ 120 \quad +120$ add 120 to each side

$L = 120$ add

$w = 210 - 1.5L$ use this equation to find w

$w = 210 - 1.5(120)$ replace L with 120

$w = 210 - 180$ multiply

$w = 30$ subtract

$(9/10)L$ this will be the new length

$= (9/10)(120)$ replace L with 120

$= 108$ multiply

$(14/15)w$ this will be the new width

$= (14/15)(30)$ replace w with 30

$= 28$ multiply

results: the new length is 108 and the new width is 28

7. A man spends $539 for sheep. He keeps 14 of the flock that he buys, and sells the remainder at an advance of $2 per head, gaining $28 by the transaction. How many sheep did he buy, and what was the cost of each?

Let s = the number of sheep that the man bought.

Let c = the original cost of each sheep

[In the end, the man had $567, (28 more than 539)]

$$\frac{539}{c} = \frac{567}{c + 2} + 14$$ here is the problem

$539(c + 2) = 567c + 14c(c + 2)$

[multiply thru by c(c + 2) and cancel as you go thru]

$539c + 1078 = 567c + 14c^2 + 28c$ multiply thru parentheses

$539c + 1078 = 14c^2 + 595c$ combine like terms

$14c^2 + 595c = 539c + 1078$ rearrange like this

$-539c \quad -539c$ subtract 539c from each side

$14c^2 + 56c = 1078$ subtract

$- 1078 \quad - 1078$ subtract 1078 from each side

$14c^2 + 56c - 1078 = 0$ subtract

$\frac{14c^2}{14} + \frac{56c}{14} - \frac{1078}{14} = \frac{0}{14}$ divide thru by 14

$c^2 + 4c - 77 = 0$ divide and cancel

$(c + 11)(c - 7) = 0$ factor

$c - 7 = 0$ set this factor equal to 0

$+ 7 \quad +7$ add 7 to each side

$c = 7$ add

$s = 539/c$ use this equation to find the number of sheep that the man bought

$s = 539/7$ replace c with 7

$s = 77$ divide

results: The man bought 77 sheep at a cost of 7 dollars per sheep.

8. A boat's crew, rowing at half their usual speed, row 3 miles downstream

and back again in 2 hours and 40 minutes.

At full speed they can go over the same course in 1 hour and 4 minutes.

Find the rate of the crew, and the rate of the current in miles per hour.

Let r = the crews usual rowing speed

Let c = the rate of the current

$$\frac{3}{(1/2)r + c} + \frac{3}{(1/2)r - c} = 2\ 2/3$$ here is the equation

$$\frac{3}{r + c} + \frac{3}{r - c} = 1\ 1/15$$ here is the 2nd equation

$$\frac{6}{r + 2c} + \frac{6}{r - 2c} = 8/3$$ on the 1st equation, multiply each fraction thru by 2, then, write 2 2/3 as an improper fraction

$$\frac{3}{r + c} + \frac{3}{r - c} = 16/15$$ write 1 1/15 as an improper fraction

$45(r - c) + 45(r + c) = 16(r - c)(r + c)$

[above, multiply thru by 15(r - c)(r + c) and cancel as you go]

$18(r - 2c) + 18(r + 2c) = 8(r - 2c)(r + 2c)$

[above, multiply thru by 3(r - 2c)(r + 2c) and cancel as you go]

$45r - 45c + 45r + 45c = 16r^2 - 16c^2$ multiply thru

$90r = 16r^2 - 16c^2$ combine like terms

$18r - 36c + 18r + 36c = 8r^2 - 32c^2$ multiply thru

$36r = 8r^2 - 32c^2$ combine like terms

$-36r = -8r^2 + 32c^2$ multiply that thru by -1

$45r = 8r^2 - 8c^2$ divide $90r = 16r^2 - 16c^2$ thru by 2

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$9r = 24c^2$ add equations

$\frac{9r}{24} = \frac{24c^2}{24}$ divide each side by 24

$(3/8)r = c^2$ reduce and cancel

$36r = 8r^2 - 32(3/8)r$ replace c^2 with $(3/8)r$

$36r = 8r^2 - 12r$ multiply

$-36r \quad - 36r$ subtract 36r from each side

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$0 = 8r^2 - 48r$ subtract

$8r^2 - 48r = 0$ rearrange like this

$\frac{8r^2}{8} - \frac{48r}{8} = \frac{0}{8}$ divide thru by 8

$r^2 - 6r = 0$ divide and cancel

$r(r - 6) = 0$ factor

$r - 6 = 0$ set this factor equal to 0

$+ 6 \quad + 6$ add 6 to each side

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$r = 6$ add

$c^2 = (3/8)r$ use this equation to find c

$c^2 = (3/8)(6)$ replace r with 6

$c^2 = 18/8$ multiply

$c^2 = 9/4$ reduce the fraction

$c = 3/2$ take square roots

results: $r = 6$ and $c = 3/2$

9. Find the sides of a rectangle whose area is unchanged

if its length is increased by 4 feet and its breadth decreased by 3 feet,

but which loses one third of its area if the length is increased by 16 feet

and the breadth decreased by 10 feet.

$Lw = (L + 4)(w - 3)$ this is the 1st equation

$(2/3)Lw = (L + 16)(w - 10)$ this is the 2nd equation

$Lw = (L + 4)(w - 3)$

$Lw = Lw - 3L + 4w - 12$ foil multiply on the 1st equation

$-Lw - Lw$ subtract Lw from each side

$0 = -3L + 4w - 12$ subtract

$+12 \quad + 12$ add 12 to each side

$12 = 4w - 3L$ add and rearrange terms

$\frac{12}{4} = \frac{4w}{4} - \frac{3L}{4}$ divide thru by 4

$3 = w - (3/4)L$ divide and cancel

$+ (3/4)L + (3/4)L$ add this to each side

$3 + (3/4)L = w$ add

$(2/3)Lw = (L + 16)(w - 10)$ this is the 2nd equation

$(2/3)(L)[3 + (3/4)L] = [L + 16][3 + (3/4)L - 10]$

[replace w with 3 + (3/4)L]

$(2/3)(L)[3 + (3/4)L] = [L + 16][(3/4)L - 7]$ combine like terms

$2L + (1/2)L^2 = (3/4)L^2 - 7L + 12L - 112$ multiply

$2L + (1/2)L^2 = (3/4)L^2 + 5L - 112$ combine like terms

$8L + 2L^2 = 3L^2 + 20L - 448$ multiply thru by 4

$- 2L^2 \quad -2L^2$ subtract $2L^2$ from each side

$8L = L^2 + 20L - 448$ subtract

$-8L \quad - 8L$ subtract 8L from each side

$0 = L^2 + 12L - 448$ subtract

$0 = (L + 28)(L - 16)$ factor

$L - 16 = 0$ set this factor equal to 0

$+ 16 \quad +16$ add 16 to each side

$L = 16$ add

$w = 3 + (3/4)(16)$ replace L with 16

$w = 3 + 12$ multiply

$w = 15$ add

results: $L = 16$ and $w = 15$