

(1.)  $|x - 7| = 3$  here is the problem

$x - 7 = 3$        $x - 7 = -3$       set equal to 3 and to -3

$+ 7$     $+7$                        $+7$     $+7$       add 7 to each side

$\frac{\quad}{x = 10}$  ;  $\frac{\quad}{x = 4}$       add

(2.)  $|3x + 2| = 5$  here is the problem

$3x + 2 = 5$        $3x + 2 = -5$       set equal to 5 and to -5

$-2$     $-2$                        $-2$     $-2$       subtract 2 from each side

$\frac{\quad}{3x = 3}$  ;  $\frac{\quad}{3x = -7}$       subtract

$\frac{\quad}{3}$     $\frac{\quad}{3}$                        $\frac{\quad}{3}$     $\frac{\quad}{3}$       divide each side by 3

$x = 1$  ;  $x = -7/3$       cancel

(3.)  $|5x + 4| = -3$

[no solution]

(4.)  $|5 - 3x| = -2$

[no solution]

(5.)  $|2x - 6| = |4 - 5x|$  here is the problem

$2x - 6 = 4 - 5x$        $2x - 6 = -4 + 5x$

$+ 6$     $+6$                        $+6$     $+6$       add 6 to each side

$\frac{\quad}{2x = 10 - 5x}$  ;  $\frac{\quad}{2x = 2 + 5x}$       add

$+ 5x$                        $+ 5x$        $-2x$                        $-2x$       add this to ea side

$\frac{\quad}{7x = 10}$  ;  $\frac{\quad}{0 = 2 + 3x}$       add

$\frac{\quad}{7}$     $\frac{\quad}{7}$                        $\frac{\quad}{3}$     $\frac{\quad}{3}$     $\frac{\quad}{3}$       divide thru by 3

$x = 10/7$  ;  $0 = (2/3) + x$       divide

$$\frac{x = 10/3}{-2/3} = \frac{-2/3}{-2/3} \quad \text{subtract } 2/3 \text{ fr ea side}$$

x          subtract

results:  $x = 10/3$        $x = -2/3$

(6.)  $|2x - 1| = |4x + 3|$                       here is the problem

$$2x - 1 = 4x + 3 \qquad 2x - 1 = -4x - 3$$

+ 1          + 1                      +1          + 1    add 1 to ea side

$$\frac{2x = 4x + 4}{-4x \quad -4x} \qquad \frac{2x = -4x - 2}{+ 4x \quad + 4x} \quad \text{add}$$

+ 4x      + 4x                      + 4x      + 4x    add this to ea side

$$\frac{-2x = 4}{6x = -2} \quad \text{add}$$

$$\frac{2}{6} \quad \frac{2}{6} \quad \text{div ea side by this}$$

$x = -2$                       ;       $x = -1/3$                       dividez

results:  $x = -2$  ;  $x = -1/3$

(7.)  $\sqrt{x - 3} = 4$                       here is the problem

$x - 3 = 16$                       square each side

+ 3      + 3                      add 3 to each side

$$\frac{x = 19}{\text{add}}$$

(8.)  $\sqrt{3x + 1} = 5$                       here is the problem

$3x + 1 = 25$                       square each side

-1      -1                      subtract 1 from each side

$$\frac{3x = 24}{\text{subtract}}$$

$$\frac{3}{3} \quad \frac{24}{3} \quad \text{divide each side by 3}$$

$x = 8$                       divide and cancel

—      —      —      —

(9.)  $\sqrt{5x} + \sqrt{3} = \sqrt{3x} - \sqrt{5}$  here is the problem  
 $\quad - \sqrt{3} \quad -\sqrt{3}$  subtract  $\sqrt{3}$  from each side

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$\sqrt{5x} = \sqrt{3x} - \sqrt{5} - \sqrt{3}$  subtract  
 $\quad - \sqrt{3x} \quad -\sqrt{3x}$  subtract this from each side

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$\sqrt{5x} - \sqrt{3x} = -\sqrt{5} - \sqrt{3}$  subtract

$\sqrt{3x} - \sqrt{5x} = \sqrt{5} + \sqrt{3}$  multiply thru by -1

$3x - 2x\sqrt{15} + 5x = \sqrt{5} + \sqrt{3}$  foil multiply

[and combine like terms]

$8x - 2x\sqrt{15} = \sqrt{5} + \sqrt{3}$  combine like terms on the left

$x(8 - 2\sqrt{15}) = \sqrt{5} + \sqrt{3}$  factor

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$\frac{8 - 2\sqrt{15}}{8 - 2\sqrt{15}} \quad \frac{8 - 2\sqrt{15}}{8 - 2\sqrt{15}}$  divide each side by this

$x = (\sqrt{5} + \sqrt{3}) / (8 - 2\sqrt{15})$  cancel

(10.)  $\sqrt{x} = 7 + \sqrt{x - 7}$  here is the problem

$x = 49 + 14\sqrt{x} - 7 + x - 7$  square each side

$-x \quad -x$  subtract x from ea side

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$0 = 49 + 14\sqrt{x} - 7 - 7$  subtract

$0 = 49 + 14\sqrt{x} - 14$  combine like terms

$0 = 35 + 14\sqrt{x}$  combine like terms

$-35 \quad -35$  subtract 35 from each side

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$$-35 = 14\sqrt{x} \quad \text{subtract}$$

result: no solution

$$(11.) \quad \sqrt{2x} - 2 = 2x - \sqrt{2} \quad \text{here is the problem}$$

Let  $u = \sqrt{2x}$  use this substitution

$$u - 2 = u^2 - \sqrt{2} \quad \text{make the substitution}$$

$$+ 2 \quad + 2 \quad \text{add 2 to each side}$$

$$\frac{u - 2 + 2}{u} = \frac{u^2 - \sqrt{2} + 2}{u^2 - \sqrt{2} + 2} \quad \text{add}$$

$$\frac{-u}{u} = \frac{-u}{u} \quad \text{subtract } u \text{ from each side}$$

$$\frac{0 = u^2 - u - \sqrt{2} + 2}{0 = u^2 - u - \sqrt{2} + 2} \quad \text{subtract}$$

$$b^2 - 4ac \quad \text{use the discriminant formula}$$

$$= (-1)^2 - 4(1)(-\sqrt{2} + 2) \quad \text{make substitutions}$$

$$= 1 + 4\sqrt{2} - 8 \quad \text{multiply thru parentheses}$$

$$= 4\sqrt{2} - 7 \quad \text{combine like terms}$$

$$= -1.34314575 \quad \text{use calculator}$$

result: no solution

$$(12.) \quad \sqrt{4x + 5} + 2\sqrt{x - 3} = 17 \quad \text{here is the problem}$$

$$4x + 5 + 4\sqrt{4x + 5}\sqrt{x - 3} + 4(x - 3) = 289 \quad \text{square each side}$$

$$4x + 5 + 4\sqrt{(4x + 5)(x - 3)} + 4x - 12 = 289 \quad \text{multiply}$$

$$8x - 7 + 4\sqrt{(4x + 5)(x - 3)} = 289 \quad \text{combine like terms}$$

$$+ 7 \quad + 7 \quad \text{add 7 to each side}$$

$$\frac{8x - 7 + 7 + 4\sqrt{(4x + 5)(x - 3)}}{8x} = \frac{296}{8x} \quad \text{add}$$

$$\frac{-8x}{-8x} = \frac{-8x}{-8x} \quad \text{subtract } 8x \text{ from each side}$$

$$\begin{array}{r}
\hline
4\sqrt{(4x + 5)(x - 3)} = 296 - 8x \quad \text{subtract} \\
\hline
\frac{\quad}{4} \quad \frac{\quad}{4} \quad \frac{\quad}{4} \quad \text{divide thru by 4} \\
\hline
\sqrt{(4x + 5)(x - 3)} = 74 - 2x \quad \text{divide} \\
(4x + 5)(x - 3) = 5476 - 296x + 4x^2 \\
4x^2 - 7x - 15 = 5476 - 296x + 4x^2 \quad \text{foil multiply} \\
-4x^2 \qquad \qquad \qquad -4x^2 \quad \text{subtract from ea side} \\
\hline
-7x - 15 = 5476 - 296x \quad \text{subtract} \\
+296x \qquad \qquad + 296x \quad \text{add this to each side} \\
\hline
289x - 15 = 5476 \quad \text{add} \\
+ 15 \quad +15 \quad \text{add 15 to each side} \\
\hline
289x = 5491 \quad \text{add} \\
\hline
\frac{\quad}{289} \quad \frac{\quad}{289} \quad \text{divide each side by 289} \\
\hline
x = 19 \quad \text{cancel}
\end{array}$$

(13.)  $\sqrt{x - 2} - \sqrt{x + 3} = 1$  here is the problem

$$\begin{array}{r}
x - 2 - 2(\sqrt{x - 2})(\sqrt{x + 3}) + x + 3 = 1 \quad \text{square each side} \\
2x + 1 - 2(\sqrt{x - 2})(\sqrt{x + 3}) = 1 \quad \text{combine like terms} \\
-1 \qquad \qquad \qquad -1 \quad \text{subtract 1 from each side} \\
\hline
2x - 2(\sqrt{x - 2})(\sqrt{x + 3}) = 0 \quad \text{subtract} \\
x - (\sqrt{x - 2})(\sqrt{x + 3}) = 0 \quad \text{divide thru by 2, cancel} \\
-x + (\sqrt{x - 2})(\sqrt{x + 3}) = 0 \quad \text{multiply thru by -1}
\end{array}$$



$$\frac{4x}{4} > \frac{20}{4} \qquad \frac{4x}{4} < \frac{-10}{4} \qquad \text{add}$$

$$x > 5 \qquad x < -2.5 \qquad \text{divide}$$

$(-\infty, -2.5) \cup (5, \infty)$  this is the interval notation

(3.)  $|-2x + 6| > 8$  here is the problem

$$-2x + 6 > 8 \qquad -2x + 6 < -8$$

$$2x - 6 < -8 \qquad 2x - 6 > 8 \qquad \text{multiply thru by -1,}$$

and change signs

$$\begin{array}{r} +6 \quad +6 \\ \hline 2x < -2 \end{array} \qquad \begin{array}{r} +6 \quad +6 \\ \hline 2x > 14 \end{array} \qquad \text{add 6 to each side}$$

$$\frac{2x}{2} < \frac{-2}{2} \qquad \frac{2x}{2} > \frac{14}{2} \qquad \text{add}$$

$(-\infty, -1) \cup (7, \infty)$  this is the interval notation

(4.)  $|3x - 4| \geq 2$  here is the problem

$$3x - 4 \geq 2 \qquad 3x - 4 \leq -2$$

$$\begin{array}{r} +4 \quad +4 \\ \hline 3x \geq 6 \end{array} ; \begin{array}{r} +4 \quad +4 \\ \hline 3x \leq 2 \end{array} \qquad \text{add 4 to each side}$$

$$\frac{3x}{3} \geq \frac{6}{3} ; \frac{3x}{3} \leq \frac{2}{3} \qquad \text{cancel}$$

$(-\infty, 2/3] \cup [2, \infty)$  this is the interval notation

(5.)  $|2x - 1| < 5$  here is the problem

$$-5 < 2x - 1 < 5$$

$$\begin{array}{r} +1 \quad + \quad 1 \quad + \quad 1 \quad \text{add 1 to each side} \\ \hline -4 < 2x < 6 \quad \text{add} \end{array}$$

$$\begin{array}{r} \frac{\quad}{2} \quad \frac{\quad}{2} \quad \frac{\quad}{2} \quad \text{divide thru by 2} \end{array}$$

$$-2 < x < 3 \quad \text{divide and cancel}$$

$(-2, 3)$  this is the interval notation

(6.)  $|2x - 3| < 4$  here is the problem

$$-4 < 2x - 3 < 4$$

$$\begin{array}{r} +3 \quad + \quad 3 \quad +3 \quad \text{add 3 to each side} \end{array}$$

$$\begin{array}{r} \hline -1 < 2x < 7 \quad \text{add} \end{array}$$

$$\begin{array}{r} \frac{\quad}{2} \quad \frac{\quad}{2} \quad \frac{\quad}{2} \quad \text{divide thru by 2} \end{array}$$

$$-1/2 < x < 7/2 \quad \text{cancel}$$

$[-1/2, 7/2]$  this is the interval notation

(7.)  $|3x - 1| < 8$

$$-8 < 3x - 1 < 8$$

$$\begin{array}{r} +1 \quad + \quad 1 \quad + \quad 1 \quad \text{add 1 to each side} \end{array}$$

$$\begin{array}{r} \hline -7 < 3x < 9 \quad \text{add} \end{array}$$

$$\begin{array}{r} \frac{\quad}{3} \quad \frac{\quad}{3} \quad \frac{\quad}{3} \quad \text{divide thru by 3} \end{array}$$

$$-7/3 < x < 3 \quad \text{divide and cancel}$$

$[-7/3, 3]$  this is the interval notation

(8.)  $|(x/3) - 7| < 5$  here is the problem

$$-5 < (x/3) - 7 < 5$$

$$-15 < x - 21 < 15 \quad \text{multiply thru by 3, cancel}$$



+ 21 + 21 + 21 add 21 to each side

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$$6 \leq x \leq 36 \quad \text{add}$$

[6,36] this is the interval notation

(9.)  $|x/3 + 2| < 4$  here is the problem

$$-4 < x/3 + 2 < 4$$

$$-12 < x + 6 < 12 \quad \text{multiply thru by 3}$$

$$-6 \quad -6 \quad -6 \quad \text{subtract 6 from each side}$$

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$$-18 < x < 6 \quad \text{subtract}$$

(-18, 6) this is the interval notation

(10.)  $|4/3 + x| \leq 2/5$  here is the problem

$$-2/5 \leq 4/3 + x \leq 2/5$$

$$-6 \leq 20 + 15x \leq 6 \quad \text{multiply thru by 15, cancel}$$

$$-20 \quad -20 \quad -20 \quad \text{subtract 20 from each side}$$

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$$-26 \leq 15x \leq -14 \quad \text{subtract}$$

$$\frac{-26}{15} \leq x \leq \frac{-14}{15} \quad \text{divide thru by 15}$$

$$-26/15 \leq x \leq -14/15 \quad \text{cancel}$$

[-26/15, -14/15] this is the interval notation

(11.)  $|2x + 5| \leq x + 3$  here is the problem

$$\text{case 1: } 2x + 5 \leq x + 3$$

$$-x \quad -x \quad \text{subtract } x \text{ from each side}$$

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$$x + 5 < 3 \quad \text{subtract}$$

$$-5 \quad -5 \quad \text{subtract 5 from each side}$$

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$$x < -2 \quad \text{subtract}$$

case 2:  $2x + 5 > -x - 3$

$$+x \quad +x \quad \text{add } x \text{ to each side}$$

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$$3x + 5 > -3 \quad \text{add}$$

$$-5 \quad -5 \quad \text{subtract 5 from each side}$$

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$$3x > -8 \quad \text{subtract}$$

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$$\frac{3x}{3} > \frac{-8}{3} \quad \text{divide each side by 3}$$

$$x > -8/3 \quad \text{cancel}$$

$$-8/3 < x < -2$$

$[-8/3, -2]$  this is the interval notation