

(1.) $\sin[(\pi/2) + t] = \cos t$ here is the problem

$\sin[(\pi/2) + t]$ start on the left side

= $\sin(\pi/2) \cos t + \cos(\pi/2) \sin t$ sum id for sine

= $(1)(\cos t) + (0)(\sin t)$ use the unit circle

= $\cos t$ multiply add

(2.) $\cos[(\pi/2) - x] = \sin x$ here is the problem

$\cos[(\pi/2) - x]$ start on the left side

= $\cos(\pi/2) \cos x + \sin(\pi/2) \sin x$ difference id for cos

= $(0)(\cos x) + (1)(\sin x)$ use the unit circle

= $\sin x$ multiply add

(3.) $\cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x$ here is the problem

$\cos^4 x - \sin^4 x$ start on the left side

= $(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$ factor

= $(\cos^2 x - \sin^2 x)(1)$ pythagorean identity

= $\cos^2 x - \sin^2 x$ multiply by 1

(4.) $1 + \sin 2x = (\sin x + \cos x)^2$ here is the problem

$(\sin x + \cos x)^2$ start on the right side

= $\sin^2 x + 2 \sin x \cos x + \cos^2 x$ square the binomial

= $1 + \sin 2x$ pythagorean id & double angle id

for sin

$$(5.) \frac{\cos^3 x - \cos x + \sin x}{\cos x} = \tan x - \sin^2 x$$

$$\frac{\cos^3 x - \cos x + \sin x}{\cos x} \quad \text{start on the left side}$$

$$= \cos^2 x - 1 + \tan x \quad \text{divide}$$

$$= (-\sin^2 x) + \tan x \quad \text{pythagorean identity}$$

$$= \tan x - \sin^2 x \quad \text{rearrange terms}$$

$$(6.) \quad \tan t + \cot t = \csc t \sec t \quad \text{here is the problem}$$

$$\tan t + \cot t \quad \text{start on the left side}$$

$$= (\csc t)(\sec t)(\sin^2 t + \cos^2 t) \quad \text{factor like this}$$

$$= (\csc t \sec t)(1) \quad \text{pythagorean identity}$$

$$= \csc t \sec t \quad \text{multiply}$$

$$(7.) \quad \frac{1}{\sec A - \tan A} = \sec A + \tan A$$

$$\frac{1}{\sec A - \tan A} \quad \text{start on the left side}$$

$$= \frac{\cos A}{1 - \sin A} \quad \text{multiply thru by cos A, cancel}$$

$$= \frac{(\cos A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)} \quad \text{multiply by this form of 1}$$

$$= \frac{\cos A + \sin A \cos A}{1 - \sin^2 A} \quad \text{multiply}$$

$$= \frac{\cos A + \sin A \cos x}{\cos^2 A} \quad \text{pythagorean identity}$$

$$= \sec A + \tan A \quad \text{divide and cancel}$$

$$(8.) \quad \frac{1 - \cos x}{\sin A} = \frac{\sin x}{1 + \cos x} \quad \text{here is the problem}$$

$$\frac{1 - \cos x}{\sin x} \quad \text{start on the left side}$$

$$(1 - \cos x)(1 + \cos x) \quad \text{multiply by this form of 1}$$

$$= \frac{(\sin x)(1 + \cos x)}{(\sin x)(1 + \cos x)}$$

$$= \frac{1 - \cos^2 x}{(\sin x)(1 + \cos x)} \quad \text{foil multiply combine like terms}$$

$$= \frac{(\sin^2 x)}{(\sin x)(1 + \cos x)} \quad \text{pythagorean identity}$$

$$= \frac{\sin x}{1 + \cos x} \quad \text{cancel}$$

$$(9.) \quad \frac{\cos A}{\csc A - 1} + \frac{\cos A}{\csc A + 1} \quad 2 \tan A$$

$$\frac{\cos A}{\csc A - 1} + \frac{\cos A}{\csc A + 1} \quad \text{start on the left side}$$

$$\frac{(\cos A)(\csc A + 1) + (\cos A)(\csc A - 1)}{\csc^2 A - 1} \quad \text{add fractions}$$

$$= \frac{(\cos A)(2 \csc A)}{\cot^2 A} \quad \text{factor and combine like terms}$$

pythagorean identity

$$= \frac{(\cos^3 A)(2 \csc A)}{\sin^2 A} \quad \text{multiply thru by } \cos^2 A$$

cancel

$$= 2 \cot^3 A \quad \text{simplify}$$

$$(10.) \quad \frac{1 - \sin x}{\cos x} = \frac{\cos x}{1 + \sin x}$$

$$\frac{1 - \sin x}{\cos x} \quad \text{start on the left side}$$

$$= \frac{(1 - \sin x)(1 + \sin x)}{(\cos x)(1 + \sin x)} \quad \text{multiply by this form of 1}$$

$$= \frac{1 - \sin^2 x}{(\cos x)(1 + \sin x)} \quad \text{foil multiply combine like terms}$$

$$= \frac{(\cos^2 x)}{(\cos x)(1 + \sin x)} \quad \text{pythagorean id}$$

$$= \frac{\cos x}{1 + \sin x} \quad \text{cancel}$$

$$(11.) \quad \sin(45 + x) + \sin(45 - x) = \sqrt{2} \cos x$$

$\sin(45 + x) + \sin(45 - x)$ start on the left side

$$= \sin 45 \cos x + \cos 45 \sin x + \sin 45 \cos x - \cos 45 \sin x$$

$$= 2 \sin 45 \cos x \quad \text{combine like terms}$$

$$= 2(\sqrt{2}/2)(\cos x) \quad \text{use the unit circle}$$

$$= \sqrt{2} \cos x \quad \text{cancel}$$

$$(12.) \quad \frac{\cos 2A}{\cos A} = \frac{1 - \tan^2 A}{\sec A}$$

$$\frac{\cos 2A}{\cos A} \quad \text{start on the left side}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A} \quad \text{double angle id for cos}$$

$$= \frac{1 - \tan^2 A}{\sec A} \quad \text{multiply thru by } \sec^2 A, \text{ cancel}$$

$$(13.) \quad \frac{\cos A}{1 - \sin A} = \frac{1 + \sin A}{\cos A}$$

$$\frac{\cos A}{1 - \sin A} \quad \text{start on the left side}$$

$$= \frac{(\cos A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)} \quad \text{multiply by this form of 1}$$

$$= \frac{(\cos A)(1 + \sin A)}{1 - \sin^2 A} \quad \text{foil multiply combine like terms}$$

$$= \frac{(\cos A)(1 + \sin A)}{\cos^2 A} \quad \text{pythagorean identity}$$

$$= (1 + \sin A)/(\cos A) \quad \text{cancel}$$

$$(14.) \quad \sin^2 A + \tan^2 A = \sec^2 A - \cos^2 A$$

$$\sin^2 A + \tan^2 A \quad \text{start on the left side}$$

$$= 1 - \cos^2 A + \tan^2 A \quad \text{pythagorean identity}$$

$$= 1 + \tan^2 A - \cos^2 A \quad \text{rearrange terms}$$

$$= \sec^2 A - \cos^2 A \quad \text{pythagorean identity}$$

$$(15.) \quad \frac{\sec x + 1}{\sec x - 1} = \cot^2 (x/2)$$

$$\cot^2 (x/2) \quad \text{start on the right side}$$

$$= \frac{\cos^2 (x/2)}{\sin^2 (x/2)} \quad \text{defintion of cot}$$

$$= \frac{1 + \cos x}{1 - \cos x} \quad \text{half angle id's for cos and sin}$$

$$= \frac{\sec x + 1}{\sec x - 1} \quad \text{multiply thru by sec x and cancel}$$

$$(16.) \quad \tan \left[\left(\frac{\pi}{4} \right) + \left(\frac{A}{2} \right) \right] = \sec A + \tan A$$

$$\tan \left[\left(\frac{\pi}{4} \right) + \left(\frac{A}{2} \right) \right] \quad \text{start on the left side}$$

$$= \frac{\tan \left(\frac{\pi}{4} \right) + \tan \left(\frac{A}{2} \right)}{1 - \left[\tan \left(\frac{\pi}{4} \right) \tan \left(\frac{A}{2} \right) \right]} \quad \text{use the sum id for tangent}$$

$$= \frac{1 + \tan \left(\frac{A}{2} \right)}{1 - \tan \left(\frac{A}{2} \right)}$$

$$\quad \text{multiply by 1}$$

$$= \frac{1 + 2 \tan \left(\frac{A}{2} \right) + \tan^2 \left(\frac{A}{2} \right)}{1 - \tan^2 \left(\frac{A}{2} \right)} \quad \begin{array}{l} \text{multiply top and bottom} \\ \text{by } 1 + \tan \left(\frac{A}{2} \right) \end{array}$$

$$= \frac{\cos^2 \left(\frac{A}{2} \right) + 2 \sin \left(\frac{A}{2} \right) \cos \left(\frac{A}{2} \right) + \sin^2 \left(\frac{A}{2} \right)}{\cos^2 \left(\frac{A}{2} \right) - \sin^2 \left(\frac{A}{2} \right)}$$

[multiply thru by $\cos^2 \left(\frac{A}{2} \right)$][and cancel]

$$= \frac{1 + \sin A}{\cos A} \quad \begin{array}{l} \text{simplify - - pythagorean id, double angle} \\ \text{id's for sin and for cos} \end{array}$$

$$= \sec A + \tan A \quad \text{divide thru by cos A}$$