(1.) A = 30; a = 6 ; b = 12

a = 6; b = 12; c = 6√3 [30-60-90 right triangle]

B = 90 ; C = 60

(2.) A = 30; a = 8; b = 12 here is the problem

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$ use the law of sines

$$\frac{\sin 30}{8} = \frac{\sin B}{12}$$ make the substitutions

12 sin 30 = 8 sin B cross multiply

$$\frac{12 \sin 30}{8} \quad \frac{8 \sin B}{8}$$ divide each side by 8

(12 sin 30)/8 = sin B cancel

B = 48.6 and B = 180 - 48.6 subtract from 180

B = 48.6 and B = 131.4 subtract

Case 1:

A + B + C = 180 use the triangle sum theorem

30 + 48.6 + C = 180 make substitutions, (use 48.6 for B)

78.6 + C = 180 combine like terms

-78.6 - 78.6 subtract 78.6 from each side

C = 101.4 subtract

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$ use the law of sines

$$\frac{\sin 101.4}{c} = \frac{\sin 30}{8}$$ make substitutions

$c \sin 30 = 8 \sin 101.4$ cross multiply

$$\frac{c \sin 30}{\sin 30} = \frac{8 \sin 101.4}{\sin 30}$$ divide each side by sin 30

c = (8 sin 101.4)/(sin 30) cancel

c = 15.68 use calculator

results: A = 30 ; B = 48.6 ; C = 101.4

a = 8 ; b = 12 ; c = 15.68

case 2:

A + B + C = 180 use the triangle sum theorem

30 + 131.4 + C = 180 make substitutions (use 131.4 for B)

161.4 + C = 180 combine like terms

-161.4 -161.4 subtract 161.4 from each side

C = 18.6 subtract

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$ use the law of sines

$$\frac{\sin 30}{8} = \frac{\sin 18.6}{c}$$ make substitutions

$c \sin 30 = 8 \sin 18.6$ cross multiply

$$\frac{c \sin 30}{\sin 30} = \frac{8 \sin 18.6}{\sin 30}$$ divide each side by sin 30

$c = (8 \sin 18.6)/(\sin 30)$ cancel

$c = 5.1$ use calculator

results: A = 30; B = 131.4 ; C = 18.6

a = 8 ; b = 12; c = 5.1

(3.) A = 30; a = 4; b = 12

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$ use the law of sines

$$\frac{\sin 30}{4} = \frac{\sin B}{12}$$ make the substitutions

$$\frac{4 \sin B}{4} = \frac{12 \sin 30}{4}$$ cross multiply; divide each side by 4

$\sin B = 3 \sin 30$ divide and cancel

$B = \arcsin(3 \sin 30)$ take the arcsin of each side

B = no solution [the sine never goes above 1]

result: no solution

(4.) A = 73 ; a = 8; b = 8

B = 73 [from geometry: opposite angles are equal, in an isosceles triangle]

$A + B + C = 180$ use the triangle sum theorem

$73 + 73 + C = 180$ make substitutions

$146 + C = 180$ combine like terms

$-146 \quad - 146$ subtract 146 from each side

$C = 34$ subtract

$c^2 = a^2 + b^2 - 2ab \cos C$ use the law of cosines

$c^2 = 8^2 + 8^2 - 2(8)(8)\cos 34$ make substitutions

$c = 7.5$ use calculator

results: A = 73 ; B = 73 ; C = 34

a = 8 ; b = 8 ; c = 7.5

(27.) A = 42; a = 5 ; b = 7 here is the problem

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$ use the law of sines

$$\frac{\sin 42}{5} = \frac{\sin B}{7}$$ make substitutions

$5 \sin B = 7 \sin 42$ cross multiply

$$\frac{5 \sin B}{5} = \frac{7 \sin 42}{5}$$ divide each side by 5

$\sin B = (7/5)\sin 42$ cancel

$B = \arcsin [(7/5) \sin 42]$ take the arcsin of each side

$B = 69.5 \qquad B = 180 - 69.5 \qquad B = 110.5$

[use calculator and subtract from 180]

case 1:

$A + B + C = 180$ use the triangle sum theorem

$42 + 69.5 + C = 180$ replace A and B with 42 & 69.5

$C + 111.5 = 180$ combine like terms

$-111.5 \quad -111.5$ subtract 111.5 from each side

$C = 68.5$ subtract

$c^2 = a^2 + b^2 - 2ab \cos C$ use the law of cosines to find c

$c^2 = (5)^2 + (7)^2 - 2(5)(7) \cos 68.5$ make substitutions

$c = 7$ use calculator

results: A = 42; B = 69.5 ; C = 68.5

a = 5 ; b = 7 ; c = 7

case 2:

$A + B + C = 180$ use the triangle sum theorem

$42 + 110.5 + C = 180$ replace A & B with 42 & 110.5

$C + 152.5 = 180$ combine like terms

$-152.5 \quad -152.5$ subtract 152.5 from each side

$C = 27.5$ subtract

$c^2 = a^2 + b^2 - 2ab \cos C$ use the law of cosines

$c^2 = (5)^2 + (7)^2 - 2(5)(7)\cos 27.5$ make substitutions

$c = 3.45$ use calculator

results: A = 42; B = 110.5 ; C = 27.5

a = 5 ; b = 7 ; c = 3.45

(28.) A = 93; a = 4 ; b = 8 here is the problem

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$ use the law of sines

$$\frac{\sin 93}{4} = \frac{\sin B}{8}$$ make substitutions

4 sin B = 8 sin 93 cross multiply

$$\frac{4 \sin B}{4} = \frac{8 \sin 93}{4}$$ divide each side by 4

sin B = 2 sin 93 divide and cancel

B = arcsin [2 sin 93] take the arcsin of each side

[no solution][the sin never goes above 1]