

Find the six trig functions for each of the following angles:

(1.) 105

$$(i.) \sin 105 = \sin 45 \cos 60 + \cos 45 \sin 60$$

[use the sum id for sine]

$$\sin 105 = (\sqrt{2}/2)(1/2) + (\sqrt{2}/2)(\sqrt{3}/2) \quad \text{use the unit circle}$$

$$\sin 105 = (\sqrt{2} + \sqrt{6})/4 \quad \text{multiply and add}$$

$$(ii.) \cos 105 = \cos 45 \cos 60 - \sin 45 \sin 60$$

[use the sum id for cosine]

$$\cos 105 = (\sqrt{2}/2)(1/2) - (\sqrt{2}/2)(\sqrt{3}/2) \quad \text{use the unit circle}$$

$$\cos 105 = (\sqrt{2} - \sqrt{6})/4$$

$$(iii.) \tan 105 = (\sin/\cos) = (\sqrt{2} + \sqrt{6})/(\sqrt{2} - \sqrt{6})$$

$$(iv.) \csc 105 = 4/(\sqrt{2} + \sqrt{6})$$

$$(v.) \sec 105 = 4/(\sqrt{2} - \sqrt{6})$$

$$(vi.) \cot 105 = (\sqrt{2} - \sqrt{6})/(\sqrt{2} + \sqrt{6})$$

(2.) 15

$$(i.) \sin 15 \quad \text{here is the problem}$$

$$= \sin (45 - 30) \quad \text{write 15 as 45 - 30}$$

$$= \sin 45 \cos 30 - \cos 45 \sin 30 \quad \text{sum id for sin}$$

$$= (\sqrt{2}/2)(\sqrt{3}/2) - (\sqrt{2}/2)(1/2) \quad \text{use the unit circle}$$

$$= (\sqrt{6} - \sqrt{2})/4 \quad \text{subtract fractions}$$

$$(ii.) \cos 15 \quad \text{here is the problem}$$

$= \cos(45 - 30)$ sum id for cos
 $= \cos 45 \cos 30 + \sin 45 \sin 30$ sum id for cos
 $= (\sqrt{2}/2)(\sqrt{3}/2) + (\sqrt{2}/2)(1/2)$ use the unit circle
 $= (\sqrt{6} + \sqrt{2})/4$ add fractions

(iii.) $\tan 15$ here is the problem
 $= (\sin 15)/(\cos 15)$ definition of tangent
 $= (\sqrt{6} - \sqrt{2})/(\sqrt{6} + \sqrt{2})$ use (i.) and (ii.)

(iv.) $\csc 15$ here is the problem
 $= 4/(\sqrt{6} - \sqrt{2})$ reciprocal of the sine

(v.) $\sec 15$ here is the problem
 $= 4/(\sqrt{6} - \sqrt{2})$ reciprocal of the cosine

(vi.) $\cot 15$ here is the problem
 $= (\sqrt{6} + \sqrt{2})/(\sqrt{6} - \sqrt{2})$

(3.) $(\pi/12)$ here is the problem
 [same as number 2 above]

(4.) 75 here is the problem

(i.) $\sin 75$ here is the problem
 $= \sin(30 + 45)$ write 75 as 30 + 45
 $= \sin 30 \cos 45 + \cos 30 \sin 45$ sum id for sine
 $= (1/2)(\sqrt{2}/2) + (\sqrt{3}/2)(\sqrt{2}/2)$ use the unit circle

$$= (\sqrt{2} + \sqrt{6})/4 \quad \text{multiply and add fractions}$$

(ii.) $\cos 75$ here is the problem

$$= \cos (30 + 45) \quad \text{write 75 as 30 + 45}$$

$$= \cos 30 \cos 45 - \sin 30 \sin 45 \quad \text{sum id for cos}$$

$$= (\sqrt{3}/2)(\sqrt{2}/2) - (1/2)(\sqrt{2}/2) \quad \text{use the unit circle}$$

$$= (\sqrt{6} - \sqrt{2})/4 \quad \text{multiply add fractions}$$

(iii.) $\tan 75$ here is the problem

$$= (\sqrt{2} + \sqrt{6})/(\sqrt{6} - \sqrt{2}) \quad \text{sin/cos [use (i.) and (ii.)]}$$

(iv.) $\csc 75$ here is the problem

$$= 1/(\sqrt{2} + \sqrt{6}) \quad \text{reciprocal of the sin}$$

(v.) $\sec 75$ here is the problem

$$= 1/(\sqrt{6} - \sqrt{2}) \quad \text{reciprocal of cos}$$

(vi.) $\cot 75$ here is the problem

$$= (\sqrt{6} - \sqrt{2})/(\sqrt{2} + \sqrt{6})$$

(9.) 195 here is the problem

(i.) $\sin 195$ here is the problem

$$= \sin (150 + 45) \quad \text{write 195 as 150 + 45}$$

$$= \sin 150 \cos 45 + \cos 150 \sin 45 \quad \text{sum id for sine}$$

$$= (1/2)(\sqrt{2}/2) + (-\sqrt{3}/2)(\sqrt{2}/2) \quad \text{use the unit circle}$$

$$= (\sqrt{2} - \sqrt{6})/4 \quad \text{multiply add}$$

(ii.) $\cos 195$ here is the problem

$$\begin{aligned}
&= \cos (150 + 45) && \text{write 195 as 150 + 45} \\
&= \cos 150 \cos 45 - \sin 150 \sin 45 && \text{sum id for cos} \\
&= (-\sqrt{3}/2)(\sqrt{2}/2) - (1/2)(\sqrt{2}/2) && \text{use the unit circle} \\
&= (-\sqrt{6} - \sqrt{2})/4 && \text{multiply subtract}
\end{aligned}$$

(iii.) $\tan 195$ here is the problem

$$= (\sqrt{2} - \sqrt{6})/(-\sqrt{6} - \sqrt{2}) \quad \text{sin/cos}$$

(iv.) $\csc 195$ here is the problem

$$= 1/(\sqrt{2} - \sqrt{6}) \quad \text{reciprocal of sine}$$

(v.) $\sec 195$ here is the problem

$$= 1/(-\sqrt{6} - \sqrt{2}) \quad \text{reciprocal of cos}$$

(vi.) $\cot 195$ here is the problem

$$= (-\sqrt{6} - \sqrt{2})/(\sqrt{2} - \sqrt{6}) \quad \text{reciprocal of tan}$$

(10.) $\sin 3A$ here is the problem

$$= \sin (2A + A) \quad \text{write 3A as 2A + A}$$

$$= \sin 2A \cos A + \cos 2A \sin A \quad \text{sum id for sine}$$

$$= 2 \sin A \cos A \cos A + (\cos^2 A - \sin^2 A)(\sin A)$$

[double angle id's for sin and cos]

$$= 2\sin A \cos^2 A + \sin A \cos^2 A - \sin^3 A \quad \text{multiply}$$

$$= 3 \sin A \cos^2 A - \sin^3 A \quad \text{combine like terms}$$