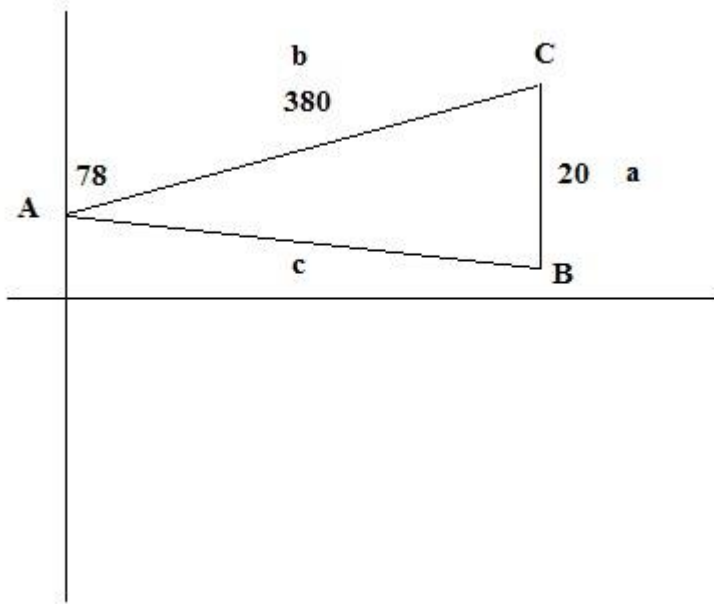


(1.) The air speed of an airplane is 380 km/hr at a bearing of 78° . The speed of the wind is 20 km/hr heading due south. Find the ground speed of the airplane as well as its direction.

Here is the diagram:



(i.) angle $C = 78$ alterate angle (geometry)

$$c^2 = a^2 + b^2 - 2ab \cos C \quad \text{use the law of cosines}$$

$$c^2 = 20^2 + 380^2 - 2(20)(380)\cos 78 \quad \text{make substitutions}$$

$$c = 376.35 \quad \text{use calculator}$$

(ii.) direction = $78 + A$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \quad \text{use the law of sines}$$

$$\frac{\sin A}{20} = \frac{\sin 78}{376.35} \quad \text{make substitutions}$$

$$376.35 \sin A = 20 \sin 78 \quad \text{cross multiply}$$

$$\frac{376.35 \sin A}{376.35} = \frac{20 \sin 78}{376.35} \quad \text{divide each side by this}$$

$$\sin A = (20 \sin 78) / (376.35) \quad \text{cancel}$$

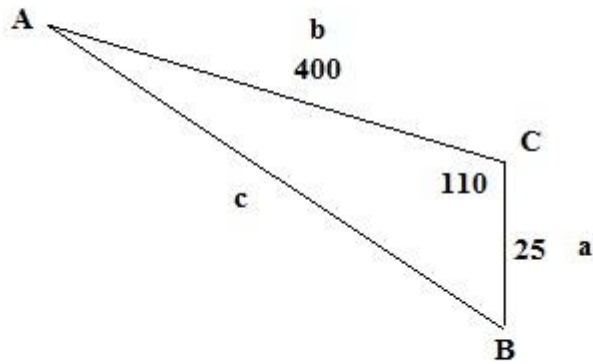
$$A = \arcsin [(20 \sin 78) / (376.35)] \quad \text{take arcsin of ea side}$$

$$A = 3 \quad \text{use calculator}$$

result: the direction will be 81° [add 3 to 78]

(2.) The air speed of an airplane is 400 km/hr at a bearing of 110° . The speed of the wind is 25 km/hr heading due south. Find the ground speed of the airplane as well as its direction.

here is the graph:



(i.) $C = 110$

$c^2 = a^2 + b^2 - 2ab \cos C$ use the law of cosines

$c^2 = 400^2 + 25^2 - 2(25)(400)\cos 110$

$c = 409$ use calculator

(ii.) direction = $110 + A$

(ii.) $\frac{\sin C}{c} = \frac{\sin A}{a}$ use the law of sines

$$\frac{\sin 110}{409} = \frac{\sin A}{25}$$

$\frac{\sin 110}{409} = \frac{\sin A}{25}$ make substitutions

$409 \sin A = 25 \sin 110$ cross multiply

$\frac{409 \sin A}{409} = \frac{25 \sin 110}{409}$ divide each side by 409

$\sin A = (25 \sin 110)/(409)$ cancel

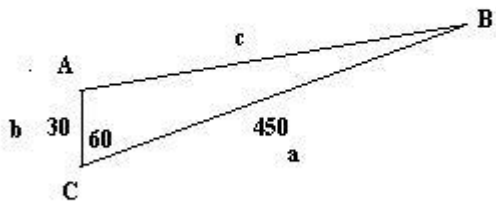
$$A = \arcsin [(25 \sin 110)/409] \quad \text{take arcsin of ea side}$$

$$A = 3.3^\circ \quad \text{use calculator}$$

$$\text{direction} = 113.3^\circ \quad \text{add 3.3 to 110}$$

- (3.) The air speed of an airplane is 450 km/hr at a bearing of 240° . The speed of the wind is 30 km/hr heading due north. Find the ground speed of the airplane as well as its direction.

here is the graph:



(i.) $c^2 = a^2 + b^2 - 2ab \cos C$ use the law of cosines

$$c^2 = 450^2 + 30^2 - 2(450)(30) \cos 60 \quad \text{make substitutions}$$

$$c = 436 \quad \text{use calculator}$$

(ii.) $\frac{\sin A}{a} = \frac{\sin C}{c}$ use the law of sines

$$\frac{\sin A}{450} = \frac{\sin 60}{436} \quad \text{make substitutions}$$

$$436 \sin A = 450 \sin 60 \quad \text{cross multiply}$$

$$\frac{436 \sin A}{436} = \frac{450 \sin 60}{436} \quad \text{divide each side by 436}$$

$$\sin A = (450 \sin 60) / (436) \quad \text{cancel}$$

$$A = \arcsin [(450 \sin 60) / (436)] \quad \text{take arcsin of each side}$$

$$A = 63^\circ \quad \text{use calculator}$$

$$A = 180 - 63 \quad \text{subtract from 180}$$

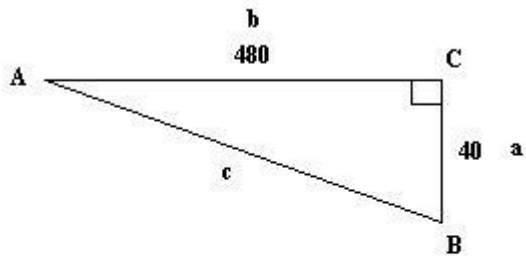
$$A = 117 \quad \text{subtract}$$

$$\text{direction} = 360 - 117 \quad \text{this will be the direction}$$

$$\text{direction} = 243 \quad \text{subtract}$$

(4.) The air speed of an airplane is 480 km/hr at a bearing of 90°. The speed of the wind is 40 km/hr heading due south. Find the ground speed of the airplane as well as its direction.

Here is the graph:



(i.) $a^2 + b^2 = c^2$ use the pythagorean theorem

$(40)^2 + (480)^2 = c^2$ make substitutions

$c = 482$ use calculator

result: $c = 482$

(ii.) $\tan A = 40/480$ use this equation to find A

$A = \arctan (40/480)$ take arctan of each side

$A = 4.8^\circ$ use calculator

direction = $90 + 4.8$ add to 90

result: direction = 94.8

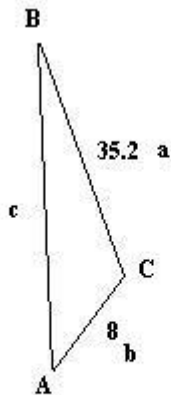
(5.) The speed of a ship is 35.2 km/hr at a bearing of

157° . The speed of the current is 8 km/hr heading in

the direction 213° .

Find the magnitude of the actual observed velocity as well as the course of the ship.

Here is the graph:



(i.) The bearing of the ship is 157, given, and the bearing of the current is 213, given.

$$C = 67 + (270 - 213)$$

$$C = 124 \quad \text{combine like terms}$$

$$c^2 = a^2 + b^2 - 2ab \cos C \quad \text{use the law of cosines}$$

$$c^2 = 35.2^2 + (8)^2 - 2(35.2)(8)\cos 124 \quad \text{make substitutions}$$

$$c = 40 \quad \text{use calculator}$$

(ii.) $\frac{\sin B}{b} = \frac{\sin C}{c}$ use the law of sines to find B

$$\frac{\sin B}{8} = \frac{\sin 124}{40} \quad \text{make substitutions}$$

$$40 \sin B = 8 \sin 124 \quad \text{cross multiply}$$

$$\frac{\quad}{40} \quad \frac{\quad}{40} \quad \text{divide each side by 40}$$

$$\sin B = (8 \sin 124)/(40) \quad \text{cancel}$$

$$B = \arcsin [(8 \sin 124)/(40)] \quad \text{take arcsin of each side}$$

$$B = 9.5 \quad \text{use calculator}$$

$$\text{direction} = 157 + 9.5 \quad \text{this will be the direction}$$

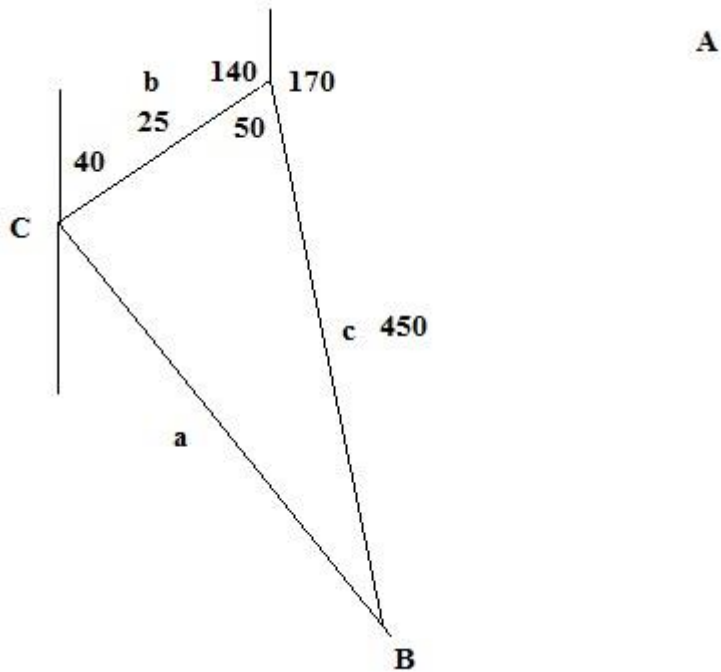
$$\text{result: direction} = 166.5 \quad (\text{add})$$

(6.) The air speed of an airplane is 380 km/hr at a bearing of 78° . The speed of the wind is 20 km/hr heading due south.

Find the ground speed of the airplane as well as its direction.

(7.) An airplane must fly at a ground speed of 450 km/hr on course 170° to be on schedule. The wind velocity is 25 km/hr in the direction 40° . Find the necessary heading and the airspeed.

here is the graph:



(i.) find a

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{use the law of cosines}$$

$$a^2 = 25^2 + 450^2 - 2(25)(450)\cos 50 \quad \text{make substitutions}$$

$$a = 434.35 \quad \text{use calculator}$$

(ii.) $\frac{\sin C}{c} = \frac{\sin A}{a}$ use the law of sines to find C

$$\frac{\sin C}{450} = \frac{\sin 50}{434.35} \quad \text{make substitutions}$$

$$434.35 \sin C = 450 \sin 50 \quad \text{cross multiply}$$

$$\frac{434.35 \sin C}{434.35} = \frac{450 \sin 50}{434.35} \quad \text{divide each side by this}$$

$$\sin C = (450 \sin 50) / (434.35) \quad \text{cancel}$$

$C = \arcsin [(450 \sin 50)/(434.35)]$ take arcsin of each side

$C = 52.5$ use calculator

$C = 180 - 52.5$ subtract from 180

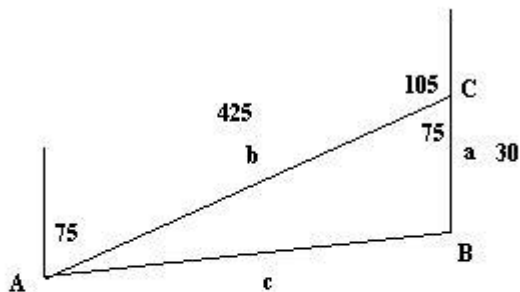
$C = 127.5$ subtract

direction = $127.5 + 40$ add 40 to find the direction

= 167.5 combine like terms

- (8.) The air speed of an airplane is 425 km/hr at a bearing of 75° . The speed of the wind is 30 km/hr heading due south. Find the ground speed of the airplane as well as its direction.

Here is the graph:



(i.) $c^2 = a^2 + b^2 - 2ab \cos C$ use the law of cosines

$$c^2 = (30)^2 + (450)^2 - 2(30)(450)\cos 75 \quad \text{make substitutions}$$

$$c = 443 \quad \text{use calculator}$$

(ii.) $\frac{\sin A}{a} = \frac{\sin C}{c}$ use the law of sines

$$\frac{\sin A}{30} = \frac{\sin 75}{443} \quad \text{make substitutions}$$

$$443 \sin A = 30 \sin 75 \quad \text{cross multiply}$$

$$\frac{443 \sin A}{443} = \frac{30 \sin 75}{443} \quad \text{divide each side by 443}$$

$$\sin A = (30 \sin 75)/(443) \quad \text{cancel}$$

$$A = \arcsin [(30 \sin 75)/(443)] \quad \text{take arcsin of ea side}$$

$$A = 3.75 \quad \text{use calculator}$$

direction = $75 + A$ this will be the direction

direction = $75 + 3.75$ make substitution

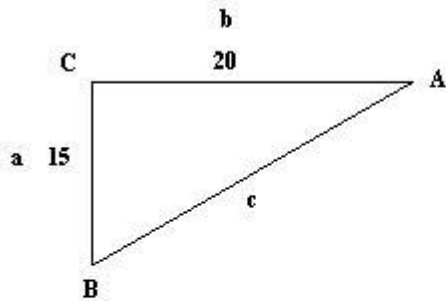
direction = 78.75 add

(9.) The speed of a boat is 15 km/hr due north.

The speed of the current is 20 km/hr heading due east.

Find the actual speed of the boat as well as its direction.

Here is the graph:



$$c^2 = a^2 + b^2 \quad \text{use the pythagorean theorem}$$

$$c^2 = 15^2 + 20^2 \quad \text{make substitutions}$$

$$c^2 = 225 + 400 \quad \text{multiply}$$

$$c^2 = 625 \quad \text{add}$$

$$c = 25 \quad \text{take square roots}$$

(ii.) $\tan B = 20/15$ use this equation to find the direction

$$B = \arctan (20/15) \quad \text{take the arctan of each side}$$

$$B = 53 \quad \text{use calculator}$$

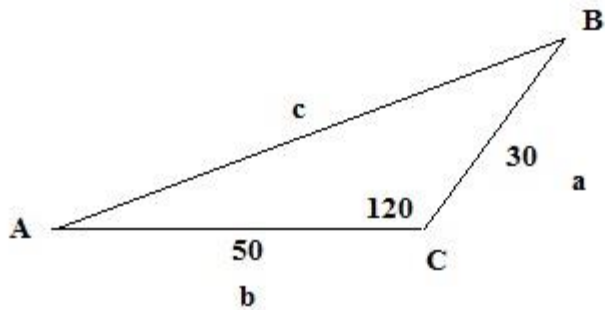
(10.) A force of 315 lbs is acting at an angle of 67° with the horizontal. What are its horizontal and vertical components?

$$x = r \cos A \quad y = r \sin A \quad \text{use these formulas}$$

$$x = 315 \cos 67 \qquad y = 315 \sin 67$$

(11.) Two force of 50 lbs and 30 lbs have an included angle of 60° . Find the magnitude and direction of their resultant.

here is the graph



(i.) $c^2 = a^2 + b^2 - 2ab \cos C$ use the law of cosines

$$c^2 = (50)^2 + (30)^2 - 2(50)(30) \cos 120$$

[make substitutions]

$$c = 70$$

(ii.) $\frac{\sin A}{a} = \frac{\sin C}{c}$ use the law of sines

$\frac{\sin A}{30} = \frac{\sin 120}{70}$ make substitutions

$70 \sin A = 30 \sin 120$ cross multiply

$\frac{70 \sin A}{70} = \frac{30 \sin 120}{70}$ divide each side by 70

$\sin A = [(30 \sin 120)/(70)]$ cancel

$A = \arcsin [(30 \sin 120)/(70)]$ take arcsin of ea side

$A = 22$ use calculator

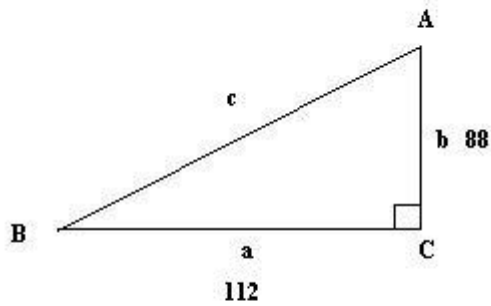
direction = $90 - 22$ subtract from 90

direction = 68

(12.) Two forces act simultaneously on a body free to move.

One force of 112 lbs is acting due east while the other of 88 lbs is acting due north. Find the magnitude and direction of their resultant.

Here is the graph:



(i.) $c^2 = a^2 + b^2$ use the pythagorean theorem

$c^2 = (112)^2 + (88)^2$ make substitutions

$c = 142.5$ use calculator

(ii.) $\tan B = b/a$ use this equation to find the
direction

$\tan B = 88/112$ make substitutions

$B = \arctan (88/112)$ take arctan of each side

$B = 38$ use calculator

direction = $90 - 38$ subtract from 90

direction = 52 subtract