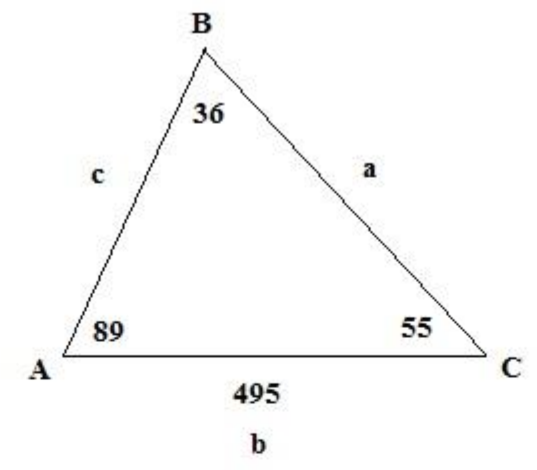
(15.) To find the distance from point A to point B across a river, a base line AC is extablished. AC is 495 meters long. Angles <BAC and <BCA are 89° and 55° respectively.

Find the distance from A to B.

Here is the diagram:



$A + B + C = 180$ use the triangle sum theorem

$89 + B + 55 = 180$ make substitutions

$B + 144 = 180$ combine like terms

$\quad -144 \quad -144$ subtract 144 from each side

$B = 36$ subtract

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$ use the law of sines

$$\frac{\sin 36}{495} = \frac{\sin 55}{c}$$ make substitutions

$c \sin 36 = 495 \sin 55$ cross multiply

$\frac{c \sin 36}{\sin 36} = \frac{495 \sin 55}{\sin 36}$ divide each side by sin 36

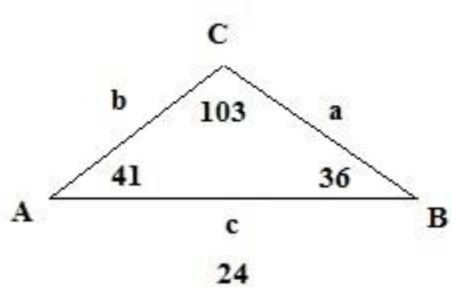
$c = (495 \sin 55)/(\sin 36)$ cancel

$c = 690$ use calculator

result: AB = 690 meters

(16.) A ship at sea is sighted from two observation posts, A and B, on shore. Points A and B are 24 km apart. The measure of the angle <A between AB and the ship is 41°. The angle at <B is 36°. Find the distance from obser - vation post A to the ship.

Here is the diagram:



$A + B + C = 180$ use the triangle sum theorem

$41 + 36 + C = 180$ make substitutions

$C + 77 = 180$ combine like terms

$-77 \quad -77$ subtract 77 from each side

$C = 103$ subtract

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$ use the law of sines

$$\frac{\sin 103}{24} = \frac{\sin 36}{b}$$ make substitutions

$b \sin 103 = 24 \sin 36$ cross multiply

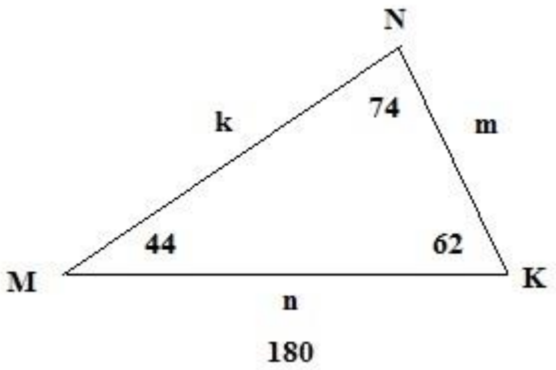
$$\frac{b \sin 103}{\sin 103} = \frac{24 \sin 36}{\sin 103}$$ divide each side by sin 103

b = (24 sin 36)/(sin 103) cancel

b = 14.5 use calculator

(17.) Two points, M and N, are separated by a swamp. A baseline MK is established on one side of the swamp. MK is 180 m in length. The angles <NMK and <MKN are measured and found to be 44° and 62° , respectively. Find the distance between M and N.

Here is the diagram:



$M + N + K = 180$ use the triangle sum theorem

$44 + N + 62 = 180$ make substitutions

$N + 106 = 180$ combine like terms

$-106 \quad -106$ subtract 106 from each side

$N = 74$ subtract

$$\frac{\sin N}{n} = \frac{\sin K}{k}$$ use the law of sines

$$\frac{\sin 74}{180} = \frac{\sin 62}{k}$$ make substitutions

$k \sin 74 = 180 \sin 62$ cross multiply

$$\frac{k \sin 74}{\sin 74} = \frac{180 \sin 62}{\sin 74}$$ divide each side by sin 74

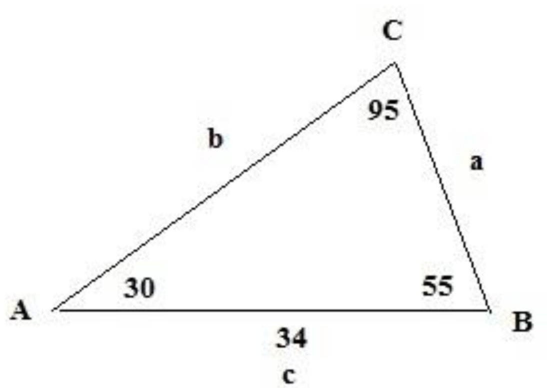
$k = (180 \sin 62)/(\sin 74)$ cancel

$k = 165$ use calculator

result: MN = 165

(18.) Two angles of a triangle are 30° and 55° and the longest side is 34 m . Find the length of the shortest side.

Here is the diagram:



A + B + C = 180   use the triangle sum theorem

30 + 55 + C = 180   make substitutions

85 + C = 180   combine like terms

-85   - 85   subtract 85 from each side

____

C = 95   subtract

$$\frac{\sin C}{} = \frac{\sin A}{}$$

$$\frac{c}{\sin 95} = \frac{a}{\sin 30}$$ use the law of sines

$$\frac{\sin 95}{34} = \frac{\sin 30}{a}$$ make the substitutions

a sin 95 = 34 sin 30 cross multiply

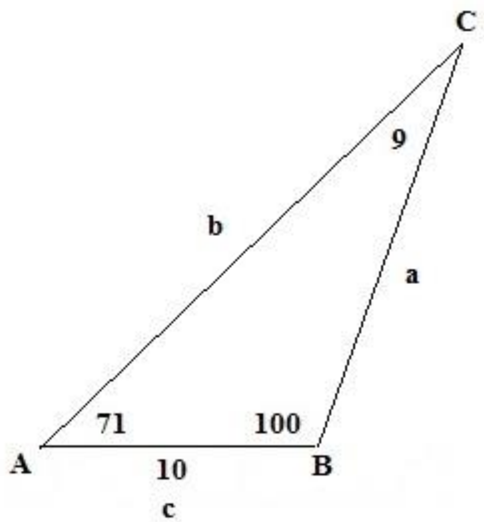
$$\frac{a \sin 95}{\sin 95} = \frac{34 \sin 30}{\sin 95}$$ divide each side by sin 95

a = (34 sin 30)/(sin 95) cancel

a = 17 use calculator

(19.) Two ranger stations located 10 km apart receive a distress call from a camper. Electronic equipment allows them to determine that the camper is at an angle of 71° from the first station and 100° from the second. Each of these angles has as one side the line segment connecting the stations. Which of the stations is closer to the camper? How far away is it from the camper?

Here is the diagram:



$A + B + C = 180$ use the triangle sum theorem

$71 + 100 + C = 180$ make substitutions

$C + 171 = 180$ combine like terms

$-171 \quad -171$ subtract 171 from each side

$C = 9$ subtract

(i.) The 2nd station is closer to the camper than the 1st station.

(ii.) $\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$ use the law of sines

$\dfrac{\sin 9}{10} = \dfrac{\sin 71}{a}$ make substitutions

$a \sin 9 = 10 \sin 71$ cross multiply

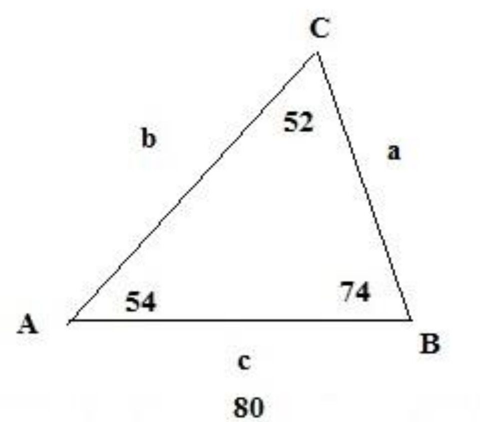
$\dfrac{a \sin 9}{\sin 9} = \dfrac{10 \sin 71}{\sin 9}$ divide each side by sin 9

$a = (10 \sin 71)/(\sin 9)$ cancel

$a = 60.4$ use calculator

(20.) A tree stands at point C across a river from point A. A baseline AB is established on one side of the river. The measure of AB is 80 m. The measure of <BAC is 54° . and that of <CBA is 74° . The angle of elevation of the top of the tree from A measures 10° . Find the height of the tree.

(i.) Here is the diagram of the triangle on the ground:



$A + B + C = 180$ use the triangle sum theorem

$54 + 74 + C = 180$ make substitutions

$C + 128 = 180$ combine like terms

-128 -128 subtract 128 from each side

C = 52 subtract

$\frac{\sin B}{b} = \frac{\sin C}{c}$ use the law of sines

$\frac{\sin 74}{b} = \frac{\sin 52}{80}$ make substitutions

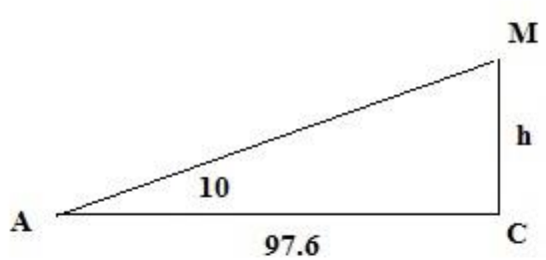
b sin 52 = 80 sin 74 cross multiply

$\frac{}{\sin 52} \quad \frac{}{\sin 52}$ divide each side by sin 52

b = (80 sin 74)/(sin 52) cancel

b = 97.6 use calculator

(ii.) Here is the diagram of the elevation of the tree:



tan 10 = h/97.6 use this trig equation

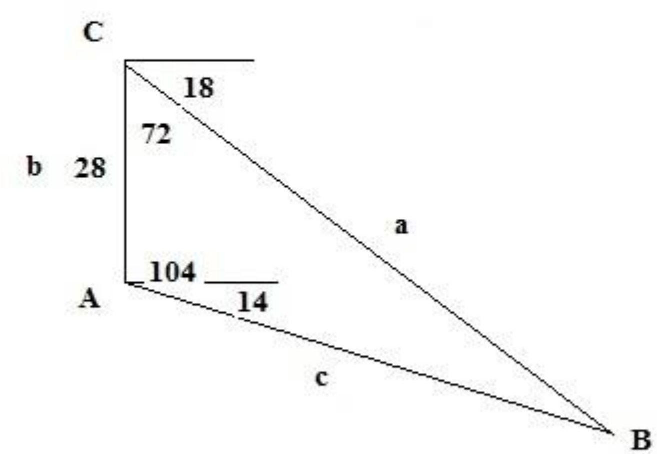
h/97.6 = tan 10 just rearrange

h = 97.6 tan 10 multiply each side by 97.6, cancel

h = 17.2   use calculator

(21.) From the top and bottom of a tower 28 m high, the angles of depression of a ship are 18° and 14°, respectively. What is the distance of the ship from the foot of the tower?

Here is the diagram:



A + B + C = 180   use the triangle sum theorem

104 + B + 72 = 180   make substitutions

B + 176 = 180   combine like terms

  -176 -176   subtract 176 from each side

B = 4   subtract

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$ use the law of sines

$$\frac{\sin 4}{28} = \frac{\sin 72}{c}$$ make substitutions

$c \sin 4 = 28 \sin 72$ cross multiply

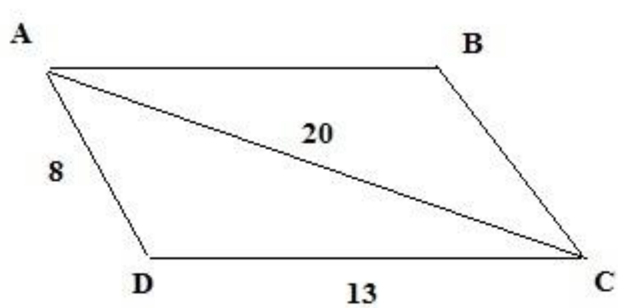
$$\frac{c \sin 4}{\sin 4} = \frac{28 \sin 72}{\sin 4}$$ divide each side by sin 4

$c = (28 \sin 72)/(\sin 4)$ cancel

$c = 381.75$ use calculator

(21b.) The lengths of two sides and one diagonal of a parallelogram are 8 m, 13 m, and 20 m, respectively. What is the measure of each angle of the parallelogram?

Here is the diagram:



$AC^2 = AD^2 + DC^2 - 2(AD)(DC)\cos D$ use the law of cosines

$20^2 = 8^2 + 13^2 - 2(8)(13) \cos D$ make substitutions

$400 = 233 - 208 \cos D$ multiply and add

$-233 \quad -233$ subtract 233 from each side

$167 = -208 \cos D$ subtract

$-208 \quad -208$ divide each side by -208

$\cos D = -167/208$ cancel and just rearrange like this

$D = \arccos(-167/208)$ take the arccos of each side

$D = 143.4$ use calculator

$180 - 143.4 = 36.6$ subtract from 180

results: <D and <B are each 143.4; and <A and <C are each 36.6

(22.) The lengths of the sides of a triangle are 8, 9 and 13 cm. Determine whether the largest angle is acute or obtuse.

$c^2 = a^2 + b^2 - 2ab \cos C$ use the law of cosines

$13^2 = (8)^2 + (9)^2 - 2(8)(9)\cos C$ make substitutions

$169 = 64 + 81 - 144 \cos C$ multiply

$169 = 145 - 144 \cos C$ combine like terms

$-145 \quad -145$ subtract 145 from each side

$24 = -144 \cos C$ subtract

$-144 \quad -144$ divide each side by -144

$-1/6 = \cos C$ reduce and cancel

result: <C is obtuse

(24.) A triangular lot has sides of 215, 185, and 125 meters.

Find the measures of the angles at its corners.

$c^2 = a^2 + b^2 - 2ab \cos C$ use the law of cosines

$215^2 = 125^2 + 185^2 - 2(125)(185)\cos C$ make substitutions

$46225 = 34225 - 46250 \cos C$ multiply and add

$-34225 \quad -34225$ subtract 34225 from each side

$12,000 = -46250 \cos C$ subtract

$$\frac{12,000}{-46250} = \frac{-46250 \cos C}{-46250}$$ divide ea side by this

$\cos C = -12000/46250$ cancel and rearrange like this

$C = \arccos(-12000/46250)$ take the arccos of each side

$C = 105$ use calculator

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$ use the law of sines

$$\frac{\sin 105}{215} = \frac{\sin B}{185}$$ make substitutions

$185 \sin 105 = 215 \sin B$ cross multiply

$$\frac{185 \sin 105}{215} = \frac{215 \sin B}{215}$$ divide each side by 215

$\sin B = (185 \sin 105)/215$ cancel

B = arcsin [(185 sin 105)/215] take the arcsin of ea side

B = 56.2 use calculator

A + B + C = 180 use the triangle sum theorem

A + 56.2 + 105 = 180 make substitutions

A + 161.2 = 180 combine like terms

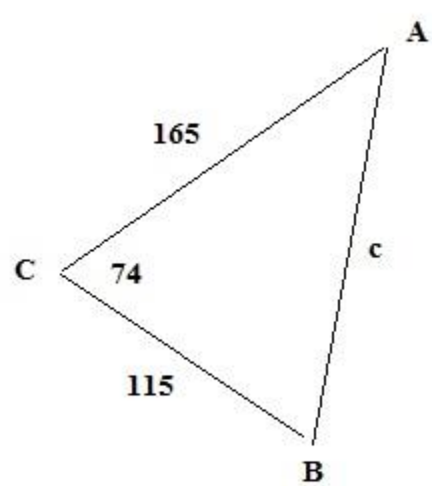
  -161.2 -161.2 subtract 161.2 from each side

A = 18.8 subtract

results: A = 18.8 ; B = 56.2 ; C = 105

(25.) From point C, both ends A and B of a railroad tunnel are visible. If AC = 165 m, BC = 115 m, and C = 74° , find AB, the length of the tunnel.

Here is the diagram:



$c^2 = a^2 + b^2 - 2ab \cos C$ use the law of sines

$c^2 = 115^2 + 165^2 - 2(115)(165)\cos 74$ make substitutions

$c^2 = 29989.56234674488$ use calculator

$c = 173$ take square roots

(26.) The distances from a boat B to two points A and C on the shore are known to be 100 m and 80 m respectively, and $\angle ABC = 55^\circ$. Find AC.

$b^2 = a^2 + c^2 - 2ac \cos B$ use the law of cosines

$b^2 = 100^2 + 80^2 - 2(100)(80) \cos 55$ make substitutions

$b^2 = 7222.777$ use calculator

$b = 85$ take square roots

(27.) The radius of a circle is 20 cm. Two radii, OX and OY, form an angle of 115°. How long is the chord XY?

$c^2 = a^2 + b^2 - 2ab \cos C$ use the law of cosines

$c^2 = 20^2 + 20^2 - 2(20)(20)\cos 115$ make substitutions

$c^2 = 1138$ use calculator

$c = 33.7$ take square roots

(28.) Two sides and a diagonal of a parallelogram are 7, 9, and 15 feet respectively. Find the measures of the angles of the parallelogram.

$c^2 = a^2 + b^2 - 2ab \cos C$ use the law of cosines

$15^2 = 7^2 + 9^2 - 2(7)(9)\cos C$ make substitutions

$225 = 130 - 126 \cos C$ multiply and add

$-225 = -130 + 126 \cos C$ multiply thru by -1

$+ 130 \quad + 130$ add 130 to each side

$95 = 126 \cos C$ add

$126 \cos C = 95$ rearrange like this

$\frac{126 \cos C}{126} = \frac{95}{126}$ divide each side by 126

$\cos C = (95/126)$ cancel

$C = \arccos (95/126)$ take arccos of each side

$C = 41$ use calculator

$180 - 41 = 139$ subtract from 180

results: the angles of the parallelogram are 139, 41, 139, & 41

(53.) A salvage ship using sonar finds the angle of depression of wreckage on the ocean floor to be 13° . The charts show that in this region the ocean floor is 35 m below the surface. How far must a diver lowered from the salvage ship travel along the ocean floor to reach the wreckage?

$\tan 13 = 35/d$ use this trig equation

$d \tan 13 = 35$ multiply each side by d, cancel

$\frac{d \tan 13}{\tan 13} = \frac{35}{\tan 13}$ divide each side by tan 13 d = 151.6 use calculator and cancel

(56.) Two travelers, driving along a highway, spot their destination in the distance at point C. They stop at the side of the road to measure angle A. (45°) They then travel 5 more miles along the road, (AB), stop again, and measure angle B. (120°) How far are the travelers now from their destination? (find AC)

A + B + C = 180 use the triangle sum theorem

45 + 120 + C = 180 make substitutions

C + 165 = 180 add

-165 -165 subtract 165 from each side

C = 15 subtract

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$ use the law of sines

$$\frac{\sin 15}{5} = \frac{\sin 120}{b}$$ make the substitutions

b sin 15 = 5 sin 120 cross multiply

$$\frac{b \sin 15}{\sin 15} = \frac{5 \sin 120}{\sin 15}$$ divide each side by sin 15

b = (5 sin 120)/(sin 15) cancel

b = 16.7 use calculator

(23.) A kite string 90 m long makes a 48° angle with the horizontal. Find, to the nearest meter, the distance from the kite to the ground.

$\sin 48 = h/90$ use this trig equation

$h/90 = \sin 48$ rearrange

$h = 90 \sin 48$ multiply each side by 90 and cancel

$h = 66.88$ use calculator

(24.) Two lighthouses at points A and B are 40 km apart. Each has visual contact with a freighter at point C.

$\angle A = 20°$ and $\angle B = 115°$. Find AC.

$A + B + C = 180$ use the triangle sum theorem

$20 + 115 + C = 180$ make substitutions

$C + 135 = 180$ combine like terms

$-135 \quad -135$ subtract 135 from each side

$C = 45$ subtract

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$ use the law of sines

$$\frac{\sin 45}{40} = \frac{\sin 115}{b}$$ make substitutions

$b \sin 45 = 40 \sin 115$ cross multiply

$$\frac{b \sin 45}{\sin 45} = \frac{40 \sin 115}{\sin 45}$$ divide each side by sin 45

$b = (4 \sin 115)/(\sin 45)$ cancel

$b = 5$ use calculator